FREE CORE NUTATION: STOCHASTIC MODELING VERSUS PREDICTABILITY

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ABSTRACT. The time series of the celestial pole offsets determined by VLBI contains the free core nutation - FCN which is pseudoharmonic variation with retrograde period of 430 days and amplitude between 0.1 and 0.3 milliarcseconds. This signal is significant at the assumed sub-milliarcsecond level of accuracy therefore needs explanation and modeling. In the first part of this study we recall our earlier results concerning the stochastic modeling of the observed FCN signal, according to the original concept of Jeffreys (1940). Then we show how the model of the autoregressive process can be applied for prediction of the observed irregular component of nutation and compare results to the extrapolation based on the sinusoidal model of the FCN.

1. INTRODUCTION

The difference between the precession and nutation observed by the *very long baseline interferometry* (VLBI) technique and that predicted by the MHB2000 model (Mathews *et al.*, 2002) contains irregular variations which are significant at the sub-milliarcsecond level of accuracy (see, e.g., Dehant *et al.*, 2003) therefore need explanation and modeling. There are at least two types of irregular motions:

1. The *free core nutation* (FCN), pseudoharmonic oscillation with retrograde period of about 430 days and variable amplitude between 0.1 and 0.3 mas (milliarcseconds) as well as variable phase.
2. Atmospheric and nontidal oceanic contributions to nutation which are not strictly harmonic but contain the broad-band variability at the level of 0.1 mas (Bizouard *et al.*, 1998; Petrov *et al.*, 1998).

These irregular motions can be modeled by different methods. Here we will focus our attention on the stochastic modeling by the *autoregressive integrated moving-average* (ARIMA) processes. First we will recall briefly our earlier results on the ARIMA modeling of the FCN, which have been discussed in a series of papers (Brzeziński, 1994; 1996; 2000; Brzeziński and Petrov, 1998; Brzeziński *et al.*, 2002). The ARIMA model is particularly suitable for determination of the parameters of the FCN resonance, the period T, the quality factor Q and the excitation power S needed to maintain the observed free celestial motion of the pole. Then we will consider the possibility of applying this model to predict the future values of the FCN signal. We will also describe first numerical experiment comparing the ARIMA prediction of the FCN to other methods.

2. FCN MODE IN EARTH ROTATION

FCN belongs to the catalogue of the solid Earth modes; see Figure 3 of Eubanks (1993). The FCN influences Earth rotation in two different ways: 1) through resonant enhancement of the amplitudes of those lunisolar nutation waves which are close to the frequency of resonance (indirect effect), and 2) it gives rise to the free oscillation of the pole in response to the irregular excitations, atmospheric, oceanic, etc. (direct effect). In addition, the FCN influences also the tidal gravity variations, but only indirect effect is detectable so far.

2.1. *Observations of the FCN*

The FCN was predicted and explained theoretically already already at the end of 19th century (Hough, 1895). Many attempts were made in the past to detect the FCN oscillation in the astrometric observations of Earth rotation, e.g. by Popov (1963), Yatskiv *et al.* (1975), but only the recent measurements by VLBI have been precise enough both to verify theoretical models of the indirect effect and to reveal the FCN signal in the celestial motion of the pole (Herring *et al.*, 1986). The FCN oscillation contributes to the observed time series of the celestial pole offsets $d\psi$, $d\varepsilon$ which are routionely provided by the International Earth Rotation Service (IERS). Several series based on the VLBI observations are available from now back to 1979, but for the purpose of tracking the FCN signal it is necessary to reject the data before 1984.

As it can be seen from Figure 1, the irregular variability of nutation consists mostly of the FCN peak which contributes more than 60% to the total power in the series. The period $T = -429.6$ days determined from our spectral analysis agrees quite well with the value $T = -430.2$ days adopted by the MHB2000 model, marked in Figure 1b by the vertical line. The amplitude of the FCN varies in time but it does not show any constant decaying trend. The maximum values, around 0.3 mas, are before 1990, then become significantly smaller in 1990-ties, and increase again after 2000.

2.2. *Modeling of the FCN*

The MHB2000 precession-nutation model (Mathews *et al.*, 2002) adopted as an official IAU standard model designated IAU2000 (IERS, 2003), does not include the FCN because it is a free motion that cannot be predicted rigorously. However, the Fortran program `IAU2000A.f` evaluating the nutation model, which is attached to Chapter 5 of the IERS Conventions 2000

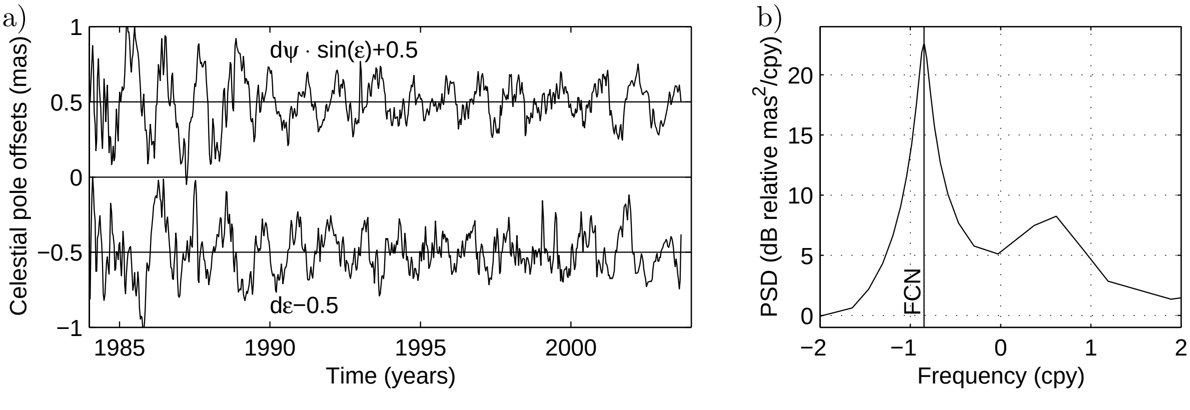

Figure 1: Celestial pole offsets observed by the VLBI (IERS combination series C04), after removal of the empirical corrections to the IAU2000 precession/nutation model a) time domain representation, b) frequency domain representation.

(*ibid.*), contains the subroutine `FCN_nut` which can be optionally used in computations. This subroutine implements the FCN model which is recommended by the IERS. The model assumes a constant dissipationless value of the FCN frequency and uses the empirical values of the sine and cosine amplitudes pre-estimated for the subsequent 2-year intervals. These amplitudes are interpolated linearly to the epoch of computation and this is the instantaneous value of the amplitude used to evaluate the FCN signal. In case when the evaluation epoch is larger than the time of the last pre-estimated value, the most recent amplitude is used in computation.

An alternative model of the observed FCN signal proposed by Shirai and Fukushima (2002) is a piecewise damped sinusoidal oscillation with a number of the excitation impulses. After assuming the values of the FCN period $T = -431$ days and the quality factor $Q = 15300$, they found that an adequate representation of the FCN signal during the period 1979 to 2000 can be obtained when assuming 4 different excitation epochs: 1989.39, 1994.47, 1994.76 and 1998.99, and determining five complex amplitudes by the least-squares method. This model has a physical explanation, namely it is an implementation of the hypothesis stating that the FCN is excited by large earthquakes.

Different approach for modelling the free oscillations in Earth rotation, originally proposed by Jeffreys (1940) as an adequate representation of the Chandler wobble, is a stochastic modeling by the ARIMA processes. The model assumes that the free signal which is subject to damping, is continuously excited by the series of independent random impulses. Such excitation corresponds to the retrograde diurnal variation in the atmospheric and oceanic angular momentum associated with the daily solar heating cycle.

The ARIMA modeling of the FCN has been discussed extensively by Brzeziński (1994, 1996, 2000), Brzeziński and Petrov (1998), Brzeziński *et al.* (2002). We proceeded according to the following scheme:

1. Introduce possible simplifications to the equation of motion.
2. Solve equation of motion analytically and perform discretization by applying the trapezoidal rule of integration.
3. Assume equidistant sampling of all variables and simple stochastic models for the excitation process and measurement noise.

If the excitation process is modeled as a white noise and the measurement errors as a random walk, then the resulting stochastic model for the observed FCN signal is

$$d_l - \varphi_1 d_{l-1} = v_l - \theta_1 v_{l-1} - \theta_2 v_{l-2}, \tag{1}$$

where $d_l = z_l - z_{l-1}$ is the first difference of the observation $z_l = P_l + n_1$ of nutation, $P_l = d\psi_l \sin\epsilon_o + i d\varepsilon_l$ denotes the nutation expressed as complex combination of the celestial pole offsets in longitude $d\psi$ and obliquity $d\varepsilon$, n_l is complex measurement noise assumed to be realization of the random walk process, the complex coefficients φ_1, θ_j are known functions of the FCN period T and quality factor Q, and v_l is a zero-mean sequence of uncorrelated complex-valued random impulses. Eq.(1) describes the ARIMA(1,1,2) model; see, e.g., (Marple, 1987) for theoretical basis. Its parameters can be determined by applying the maximum likelihood algorithm to the time series of observations.

The ARIMA model expressed by eq.(1) which follows from the physics of the FCN, provides the optimum representation in a sense of the criterion of parsimony: it is fully describes by the three complex coefficients φ_1, θ_1, θ_2 and one real coefficient expressing the standard deviation of the driving noise v_l. An equivalent from the point of view of mathematics, representation of the ARIMA process is provided by the pure *autoregressive* (AR) model

$$z_l - \varphi_1 z_{l-1} - \varphi_2 z_{l-2} - \ldots\ldots - \varphi_p z_{l-p} = v_l. \tag{2}$$

This model involves usually more parameters than the optimum ARIMA counterpart, eq.(1). In case of the FCN signal shown in Figure 1a, with 10-day sampling, we found the optimum AR order $p = 21$ (see next section for details). That means the number of parameters increased seven times. But such a pure AR model offers important advantages for our applications. First, there exist simple algorithms for determining its parameters, the coefficients φ_j, $j = 1, \ldots, p$, and standard deviation of the driving noise v_l. In the computations described in the next section we applied the least-squares version of the maximum entropy method (MEM) with the Akkaike final prediction error (FPE) criterion for finding the optimum order p; see (Brzeziński, 1995) for computational details. Second, the application of the AR model for prediction is straightforward. One needs only to replace in eq.(2) the unknown future random impulses v_l by their expected values, that is by zeros. Finally, this model takes into account not only the FCN signal but also other irregular components of the observed time series, such as that expressed by the broad peak at prograde frequencies between 0 and 1 cpy, shown in Figure 1b.

3. PREDICTION OF THE IRREGULAR VARIATIONS IN NUTATION

We used in computations the IERS combination series C04 related to the IAU2000 precession-nutation model (file `EOPC04_IAU2000.62-now` downloaded from the following website address `http://hpiers.obspm.fr/eoppc/eop/eopC04`). The series used here contains 7192 daily values of the celestial pole offsets $X = d\psi \sin \varepsilon_o$, $Y = d\varepsilon$ spanning the period between 1984.0 and 2003.7. First, we removed from the input series the model comprising the linear trend and corrections to the important nutation terms. We found by the best least-squares fit the following corrections which are in some cases surprisingly large.

1-st order polynomial		$d\psi \sin \varepsilon_o$	$d\varepsilon$
	constant (μas)	59 ± 6	32 ± 7
	slope (μas/yr)	8 ± 1	9 ± 1
Periodical terms (μas)		retrograde	prograde
	18.6 yr	28 ± 8	68 ± 6
	9.3 yr	28 ± 6	33 ± 6
	1.0 yr	5 ± 6	10 ± 6
	0.5 yr	19 ± 7	9 ± 6

Then, we smoothed the residual nutation series by the Gaussian filter with full width at a half of maximum equal to 20 days, and interpolated to the equidistant epochs with sampling interval 10 days. The reduced nutation series is shown in Figure 1a. Its MEM power spectrum computed for the AR order $p = 21$ (cf. eq.(2)) shows the FCN peak containing more than 60% power of the series. The MEM spectral analysis yields the FCN period of $T = -429.6$ days, the quality factor $Q = 2995$ and the mean amplitude, that is the square root of the power of oscillation, $A = 179$ μas. If we compute the FCN amplitude assuming purely sinusoidal model over the entire time interval 1984.0 to 2003.7, the result is 107 ± 6 μas for the retrograde component and 11 ± 6 μas for the prograde one.

Next, we computed the one year-long forecasts of the residual nutation series using two different methods. The first method, designated below as LS prediction, corresponds to the FCN model recommended by the IERS. It consists in fitting to the last two years of data the model comprising a sum of the complex sinusoid with period of -430 days and the constant, which is then extrapolated into the future. The other method is the AR prediction with the coefficient estimated by using the last 8 years of data. The optimum AR order was determined by finding the first larger than 20 local minimum of the FPE value. We also considered as a reference the third method assuming constant prediction, where the constant was determined

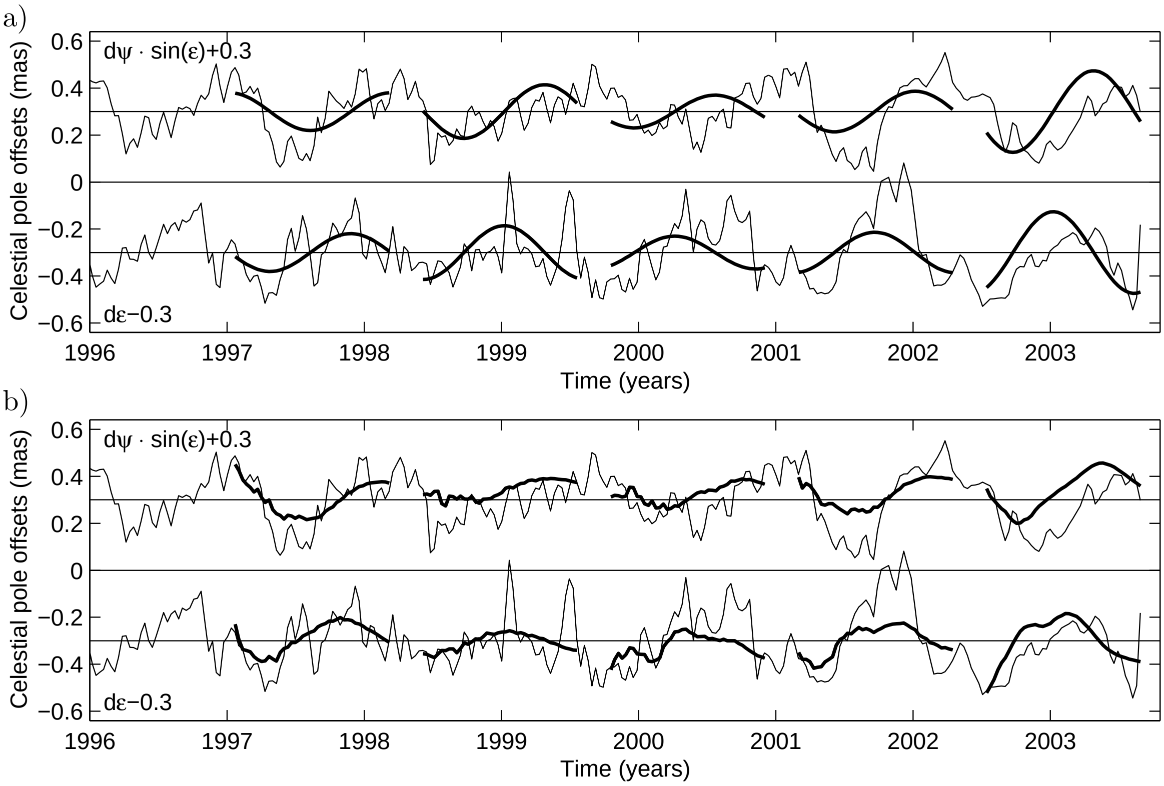

Figure 2: One year predictions (thick lines) of the observed nutation residuals (thin line), computed at different starting epochs a) by the least squares extrapolation of the model: constant plus sinusoid with period 430 days, b) by the autoregressive prediction.

as a mean value over the last 2.5 years of data, that is roughly two full FCN cycles. The mean prediction error for t days in future was computed as a difference between prediction and true value averaged over the whole set of possible starting prediction times. Figure 2 shows sample predictions by the first two methods, computed for different starting points, while the mean prediction errors for t between 10 days and 1 year are compared in Figure 3. For other prediction methods applied to the observations of polar motion see the paper by Kosek *et al.* (this volume) and the references therein.

From the inspection of Figures 2 and 3 we can draw several conclusions. All predictions are generally worse for $Y = d\varepsilon$ which indicate the higher noise contents than in $X = d\psi \sin \varepsilon_o$. The mean error of the constant prediction does not depend on the prediction time, while in case of the sinusoidal model the error increases linearly. As could be expected, the best results are obtained by the AR model. The prediction error growths rapidly for t between 10 and 50 days, then increases linearly, and finally becomes almost constant. Again, there is a difference between the X and Y components of nutation. In case of the X component the initial growth of error is slower, and the stability is reached already for t equal to about 200 days and at the level of 0.10 mas. In case of Y, the corresponding quantities are 270 days and about 0.12 mas. The advantage over the sinusoidal extrapolation is particularly well seen for short and long prediction times. As it can be seen from Figure 2, the reason for better performance of the AR prediction is that the underlying stochastic model does not only attempt to express the FCN but also other fine features of the irregular variability of nutation.

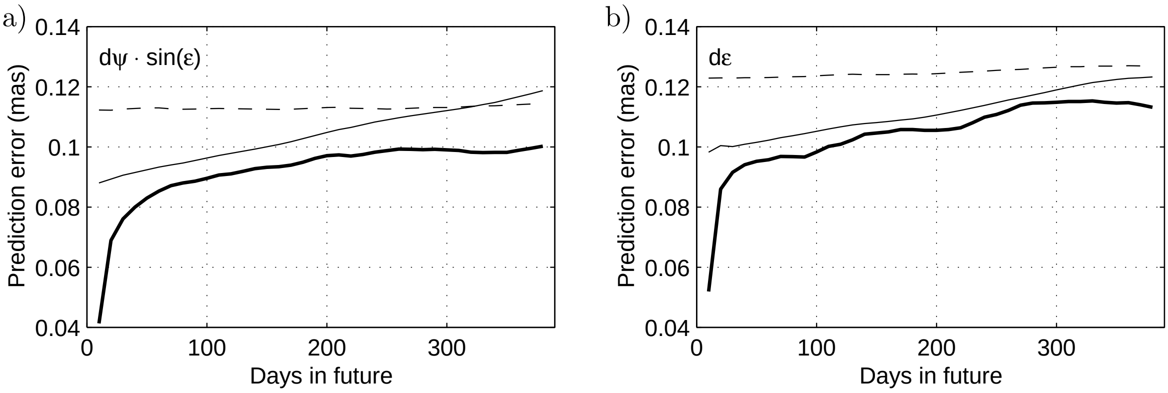

Figure 3: The mean prediction error of the nutation angles computed from the least-squares extrapolation of constant (dashed line), of constant plus sinusoid with period of 430 days (thin solid line) and from the autoregressive prediction (thick solid line).

4. SUMMARY AND CONCLUSIONS

Modeling the FCN signal in the time series of the celestial pole offsets observed by VLBI, is an important task of the submilliarcsecond astrometry. Its variability (Figure 1) is typical for the randomly excited free oscillator with damping. A similar observation concerning the Chandler wobble led Jeffreys (1940) to the concept of describing the free motion as a realization of the stochastic process, the ARIMA process. We followed this concept and applied the ARIMA model for description of the FCN; see, e.g., (Brzeziński, 1996) for details.

The application of ARIMA processes in the investigations concerning the FCN offers several advantages. Such a model has a good physical explanation since it can be derived directly from the equation of motion. The underlying assumption is that the excitation function behaves as a random process, which is reasonable as far as we consider the influence of the atmosphere and nontidal oceanic variability on nutation. The application of the ARIMA model enables determination of the parameters of the FCN mode (Brzeziński and Petrov, 1998) which is independent from the estimation based on the indirect effect (Mathews *et al.*, 2002). The ARIMA model can be also used for time domain comparison between the FCN signal and the atmospheric and/or oceanic excitation, but so far we have not the adequate estimate of the excitation function (Petrov *et al.*, 1997).

In this research we considered the application of the AR model for prediction of the irregular component of nutation including the FCN signal. Application to the observed time series of the celestial pole offsets demonstrated clear advantage of the AR-based prediction over the extrapolation of the sinusoidal model. This advantage is particularly large for short term predictions, up to about 1 month. For longer prediction times, between 1/2 and 1 year, the AR model yields errors of about 0.10 mas for $X = d\psi \sin \varepsilon_o$ and 0.11 mas for $Y = d\varepsilon$.

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